Main Ideas
- Write quadratic equations in intercept form.
- Solve quadratic equations by factoring.

New Vocabulary
intercept form
FOIL method

The intercept form of a quadratic equation is
\[ y = a(x - p)(x - q) \]
In the equation, \( p \) and \( q \) represent the \( x \)-intercepts of the graph corresponding to the equation. The intercept form of the equation shown in the graph is
\[ y = 2(x - 1)(x + 2) \]
The \( x \)-intercepts of the graph are 1 and \(-2\). The standard form of the equation is
\[ y = 2x^2 + 2x - 4 \]

**Intercept Form** Changing a quadratic equation in intercept form to standard form requires the use of the FOIL method. The FOIL method uses the Distributive Property to multiply binomials.

**KEY CONCEPT**

The product of two binomials is the sum of the products of \( F \) the first terms, \( O \) the outer terms, \( I \) the inner terms, and \( L \) the last terms.

To change \( y = 2(x - 1)(x + 2) \) to standard form, use the FOIL method to find the product of \((x - 1)\) and \((x + 2)\), \(x^2 + x - 2\), and then multiply by 2. The standard form of the equation is \( y = 2x^2 + 2x - 4 \).

You have seen that a quadratic equation of the form \((x - p)(x - q) = 0\) has roots \( p \) and \( q \). You can use this pattern to find a quadratic equation for a given pair of roots.

**EXAMPLE** Write an Equation Given Roots

Write a quadratic equation with \( \frac{1}{2} \) and \(-5\) as its roots. Write the equation in the form \( ax^2 + bx + c = 0 \), where \( a \), \( b \), and \( c \) are integers.

\[
(x - p)(x - q) = 0 \quad \text{Write the pattern.}
\]
\[
\left( x - \frac{1}{2} \right) \left[ x - (-5) \right] = 0 \quad \text{Replace } p \text{ with } \frac{1}{2} \text{ and } q \text{ with } -5.
\]
\[
\left( x - \frac{1}{2} \right) (x + 5) = 0 \quad \text{Simplify.}
\]
\[
x^2 + \frac{9}{2}x - \frac{5}{2} = 0 \quad \text{Use FOIL.}
\]
\[
2x^2 + 9x - 5 = 0 \quad \text{Multiply each side by 2 so that } b \text{ and } c \text{ are integers.}
\]

1. Write a quadratic equation with \( -\frac{1}{3} \) and 4 as its roots. Write the equation in standard form.
**Solve Equations by Factoring** In the last lesson, you learned to solve a quadratic equation by graphing. Another way to solve a quadratic equation is by factoring an equation in standard form. When an equation in standard form is factored and written in intercept form $y = a(x - p)(x - q)$, the solutions of the equation are $p$ and $q$.

The following factoring techniques, or patterns, will help you factor polynomials. Then you can use the Zero Product Property to solve equations.

<table>
<thead>
<tr>
<th>Factoring Technique</th>
<th>General Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greatest Common Factor (GCF)</td>
<td>$a^3b^2 - 3ab^2 = ab^2(a^2 - 3)$</td>
</tr>
<tr>
<td>Difference of Two Squares</td>
<td>$a^2 - b^2 = (a + b)(a - b)$</td>
</tr>
</tbody>
</table>
| Perfect Square Trinomials | $a^2 + 2ab + b^2 = (a + b)^2$  
$a^2 - 2ab + b^2 = (a - b)^2$ |
| General Trinomials | $acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$ |

The FOIL method can help you factor a polynomial into the product of two binomials. Study the following example.

\[
(ax + b)(cx + d) = \overset{F}{ax} \cdot \overset{O}{cx} + \overset{I}{ax} \cdot \overset{L}{d} + \overset{I}{b} \cdot \overset{O}{cx} + \overset{F}{b} \cdot \overset{L}{d}
\]

\[
= acx^2 + (ad + bc)x + bd
\]

Notice that the product of the coefficient of $x^2$ and the constant term is $abcd$. The product of the two terms in the coefficient of $x$ is also $abcd$.

**EXAMPLE**

Factor each polynomial.

**a.** $5x^2 - 13x + 6$

To find the coefficients of the $x$-terms, you must find two numbers with a product of $5 \cdot 6$ or $30$, and a sum of $-13$. The two coefficients must be $-10$ and $-3$ since $(-10)(-3) = 30$ and $-10 + (-3) = -13$.

Rewrite the expression using $-10x$ and $-3x$ in place of $-13x$ and factor by grouping.

\[
5x^2 - 13x + 6 = 5x^2 - 10x - 3x + 6
\]

\[
= (5x^2 - 10x) + (-3x + 6)
\]

\[
= 5x(x - 2) - 3(x - 2)
\]

\[
= (5x - 3)(x - 2)
\]

**b.** $m^6 - n^6$

\[
m^6 - n^6 = (m^3 + n^3)(m^3 - n^3)
\]

\[
= (m + n)(m^2 - mn + n^2)(m - n)(m^2 + mn + n^2)
\]

The difference of two squares should always be done before the difference of two cubes. This will make the next step of the factorization easier.

**CHECK Your Progress**

2A. $3xy^2 - 48x$  
2B. $c^3d^3 + 27$
Solving quadratic equations by factoring is an application of the Zero Product
Property.

**Key Concept**

**Zero Product Property**

**Words**
For any real numbers \(a\) and \(b\), if \(ab = 0\), then either \(a = 0\), \(b = 0\), or both \(a\) and \(b\) equal zero.

**Example**
If \((x + 5)(x - 7) = 0\), then \(x + 5 = 0\) or \(x - 7 = 0\).

---

**Example**

**Two Roots**

Solve \(x^2 = 6x\) by factoring. Then graph.

\[
x^2 = 6x \quad \text{Original equation}
\]

\[
x^2 - 6x = 0 \quad \text{Subtract } 6x \text{ from each side.}
\]

\[
x(x - 6) = 0 \quad \text{Factor the binomial.}
\]

\[
x = 0 \quad \text{or} \quad x - 6 = 0 \quad \text{Zero Product Property}
\]

\[
x = 6 \quad \text{Solve the second equation.}
\]

The solution set is \(\{0, 6\}\).

To complete the graph, find the vertex. Use the equation for the axis of
symmetry.

\[
x = \frac{-b}{2a} \quad \text{Equation of the axis of symmetry}
\]

\[
x = \frac{-(-6)}{2(1)} = \frac{6}{2} = 3 \quad \text{Simplify.}
\]

Therefore, the \(x\)-coordinate of the vertex is 3.

Substitute 3 into the equation to find the \(y\)-value.

\[
y = x^2 - 6x \quad \text{Original equation}
\]

\[
y = 3^2 - 6(3) \quad x = 3
\]

\[
y = 9 - 18 \quad \text{Simplify.}
\]

\[
y = -9 \quad \text{Subtract.}
\]

The vertex is at \((3, -9)\). Graph the \(x\)-intercepts \((0, 0)\) and \((6, 0)\) and the
vertex \((3, -9)\), connecting them with a smooth curve.

**Check Your Progress**

Solve each equation by factoring. Then graph.

3A. \(3x^2 = 9x\) 
3B. \(6x^2 = 1 - x\)

---

**Example**

**Double Root**

Solve \(x^2 - 16x + 64 = 0\) by factoring.

\[
x^2 - 16x + 64 = 0 \quad \text{Original equation}
\]

\[
(x - 8)(x - 8) = 0 \quad \text{Factor.}
\]

\[
x - 8 = 0 \quad \text{or} \quad x - 8 = 0 \quad \text{Zero Product Property}
\]

\[
x = 8 \quad \text{Solve each equation.}
\]

The solution set is \(\{8\}\). (continued on the next page)
CHECK The graph of the related function, \( f(x) = x^2 - 16x + 64 \), intersects the \( x \)-axis only once. Since the zero of the function is 8, the solution of the related equation is 8.

Solve each equation by factoring.
4A. \( x^2 + 12x + 36 = 0 \)  
4B. \( x^2 - 25 = 0 \)

Example 1 (p. 253) Write a quadratic equation with the given root(s). Write the equation in standard form.
1. \(-4, 7\)  
2. \(1, \frac{4}{3}\)  
3. \(-\frac{3}{5}, \frac{1}{3}\)

Example 2 (p. 254) Factor each polynomial.
4. \(x^3 - 27\)  
5. \(4xy^2 - 16x\)  
6. \(3x^2 + 8x + 5\)

Examples 3, 4 (pp. 255–256) Solve each equation by factoring. Then graph.
7. \(x^2 - 11x = 0\)  
8. \(x^2 + 6x - 16 = 0\)  
9. \(4x^2 - 13x = 12\)  
10. \(x^2 - 14x = -49\)  
11. \(x^2 + 9 = 6x\)  
12. \(x^2 - 3x = -\frac{9}{4}\)

Write a quadratic equation in standard form for each graph.
13.  
14.  

Write a quadratic equation in standard form with the given roots.
15. 4, −5  
16. −6, −8

Factor each polynomial.
17. \(x^2 - 7x + 6\)  
18. \(x^2 + 8x - 9\)  
19. \(3x^2 + 12x - 63\)  
20. \(5x^2 - 80\)

Solve each equation by factoring. Then graph.
21. \(x^2 + 5x - 24 = 0\)  
22. \(x^2 - 3x - 28 = 0\)  
23. \(x^2 = 25\)  
24. \(x^2 = 81\)  
25. \(x^2 + 3x = 18\)  
26. \(x^2 - 4x = 21\)  
27. \(-2x^2 + 12x - 16 = 0\)  
28. \(-3x^2 - 6x + 9 = 0\)  
29. \(x^2 + 36 = 12x\)  
30. \(x^2 + 64 = 16x\)

31. NUMBER THEORY Find two consecutive even integers with a product of 224.
32. **PHOTOGRAPHY** A rectangular photograph is 8 centimeters wide and 12 centimeters long. The photograph is enlarged by increasing the length and width by an equal amount in order to double its area. What are the dimensions of the new photograph?

Solve each equation by factoring.

33. \(3x^2 = 5x\)
34. \(4x^2 = -3x\)
35. \(4x^2 + 7x = 2\)
36. \(4x^2 - 17x = -4\)
37. \(4x^2 + 8x = -3\)
38. \(6x^2 + 6 = -13x\)
39. \(9x^2 + 30x = -16\)
40. \(16x^2 - 48x = -27\)

41. Find the roots of \(x(x + 6)(x - 5) = 0\).
42. Solve \(x^3 = 9x\) by factoring.

Write a quadratic equation with the given graph or roots.

43. \(\frac{2}{3}, \frac{3}{4}\)
44. \(-\frac{3}{2}, -\frac{4}{5}\)

47. **DIVING** To avoid hitting any rocks below, a cliff diver jumps up and out. The equation \(h = -16t^2 + 4t + 26\) describes her height \(h\) in feet \(t\) seconds after jumping. Find the time at which she returns to a height of 26 feet.

**FORESTRY** For Exercises 48 and 49, use the following information.

Lumber companies need to be able to estimate the number of board feet that a given log will yield. One of the most commonly used formulas for estimating board feet is the *Doyle Log Rule*, 

\[ B = \frac{L}{16}(D^2 - 8D + 16) \]

where \(B\) is the number of board feet, \(D\) is the diameter in inches, and \(L\) is the length of the log in feet.

48. Rewrite Doyle’s formula for logs that are 16 feet long.
49. Find the root(s) of the quadratic equation you wrote in Exercise 48. What do the root(s) tell you about the kinds of logs for which Doyle’s rule makes sense?

50. **FIND THE ERROR** Lina and Kristin are solving \(x^2 + 2x = 8\). Who is correct? Explain your reasoning.

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**Real-World Link**

A board foot is a measure of lumber volume. One piece of lumber 1 foot long by 1 foot wide by 1 inch thick measures one board foot.

Source: www.wood-worker.com

**EXTRA PRACTICE**

See pages 900, 930.

**Math Online**

Self-Check Quiz at algebra2.com
51. **OPEN ENDED** Choose two integers. Then write an equation with those roots in standard form. How would the equation change if the signs of the two roots were switched?

52. **CHALLENGE** For a quadratic equation of the form \((x - p)(x - q) = 0\), show that the axis of symmetry of the related quadratic function is located halfway between the x-intercepts \(p\) and \(q\).

53. **Writing in Math** Use the information on page 253 to explain how to solve a quadratic equation using the Zero Product Property. Explain why you cannot solve \(x(x + 5) = 24\) by solving \(x = 24\) and \(x + 5 = 24\).

54. **ACT/SAT** Which quadratic equation has roots \(\frac{1}{2}\) and \(\frac{1}{3}\)?
   - A  \(5x^2 - 5x - 2 = 0\)
   - B  \(5x^2 - 5x + 1 = 0\)
   - C  \(6x^2 + 5x - 1 = 0\)
   - D  \(6x^2 - 5x + 1 = 0\)

55. **REVIEW** What is the solution set for the equation \(3(4x + 1)^2 = 48\)?
   - F  \(\left\{\frac{5}{4}, \frac{3}{4}\right\}\)
   - H  \(\left\{\frac{15}{4}, \frac{-17}{4}\right\}\)
   - G  \(\left\{-\frac{5}{4}, \frac{3}{4}\right\}\)
   - J  \(\left\{\frac{1}{3}, \frac{-4}{3}\right\}\)

**Spiral Review**

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. (Lesson 5-2)

56. \(0 = -x^2 - 4x + 5\)
57. \(0 = 4x^2 + 4x + 1\)
58. \(0 = 3x^2 - 10x - 4\)

59. Determine whether \(f(x) = 3x^2 - 12x - 7\) has a maximum or a minimum value. Then find the maximum or minimum value. (Lesson 5-1)

60. **CAR MAINTENANCE** Vince needs 12 quarts of a 60% anti-freeze solution. He will combine an amount of 100% anti-freeze with an amount of a 50% anti-freeze solution. How many quarts of each solution should be mixed to make the required amount of the 60% anti-freeze solution? (Lesson 4-8)

Write an equation in slope-intercept form for each graph. (Lesson 2-4)

61.  

62.  

**GET READY for the Next Lesson**

**PREREQUISITE SKILL** Name the property illustrated by each equation. (Lesson 1-2)

63. \(2x + 4y + 3z = 2x + 3z + 4y\)
64. \(3(6x - 7y) = 3(6x) + 3(-7y)\)
65. \((3 + 4) + x = 3 + (4 + x)\)
66. \((5x)(-3y)(6) = (-3y)(6)(5x)\)
Consider \(2x^2 + 2 = 0\). One step in the solution of this equation is \(x^2 = -1\). Since there is no real number that has a square of \(-1\), there are no real solutions. French mathematician René Descartes (1596–1650) proposed that a number \(i\) be defined such that \(i^2 = -1\).

### Square Roots and Pure Imaginary Numbers

A square root of a number \(n\) is a number with a square of \(n\). For example, 7 is a square root of 49 because \(7^2 = 49\). Since \((-7)^2 = 49\), \(-7\) is also a square root of 49. Two properties will help you simplify expressions that contain square roots.

**Simplified square root expressions** do not have radicals in the denominator, and any number remaining under the square root has no perfect square factor other than 1.

**EXAMPLE**

**Properties of Square Roots**

Simplify.

1. \(\sqrt{50}\)
   \[
   \sqrt{50} = \sqrt{25 \cdot 2} \\
   = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2} \\
   
   1A. \(\sqrt{45}\)
   
   1B. \(\sqrt{\frac{32}{81}}\)

Since \(i\) is defined to have the property that \(i^2 = -1\), the number \(i\) is the principal square root of \(-1\); that is, \(i = \sqrt{-1}\). \(i\) is called the **imaginary unit**. Numbers of the form \(3i\), \(-5i\), and \(i\sqrt{2}\) are called **pure imaginary numbers**. Pure imaginary numbers are square roots of negative real numbers. For any positive real number \(b\), \(\sqrt{-b^2} = \sqrt{b^2} \cdot \sqrt{-1} = bi\).
EXAMPLE

Square Roots of Negative Numbers

Simplify.

a. \( \sqrt{-18} \)
\[
\begin{align*}
\sqrt{-18} &= \sqrt{-1 \cdot 3^2 \cdot 2} \\
&= \sqrt{-1} \cdot \sqrt{3^2} \cdot \sqrt{2} \\
&= i \cdot 3 \cdot \sqrt{2} \text{ or } 3i\sqrt{2}
\end{align*}
\]

b. \( \sqrt{-125x^5} \)
\[
\begin{align*}
\sqrt{-125x^5} &= \sqrt{-1 \cdot 5^2 \cdot x^4 \cdot 5x} \\
&= \sqrt{-1} \cdot \sqrt{5^2} \cdot \sqrt{x^4} \cdot \sqrt{5x} \\
&= i \cdot 5 \cdot x^2 \cdot \sqrt{5x} \text{ or } 5ix^2\sqrt{5x}
\end{align*}
\]

CHECK Your Progress

2A. \( \sqrt{-27} \)

\[ \sqrt{-1 \cdot 3^3} = i \cdot 3 \cdot \sqrt{3} \]

2B. \( \sqrt{-216y^4} \)

\[ \sqrt{-1 \cdot 3^2 \cdot 2^2 \cdot y^4} = i \cdot 3 \cdot 2 \cdot y^2 \sqrt{2} \]

The Commutative and Associative Properties of Multiplication hold true for pure imaginary numbers.

EXAMPLE

Products of Pure Imaginary Numbers

Simplify.

a. \(-2i \cdot 7i \)
\[
\begin{align*}
-2i \cdot 7i &= -14i^2 \\
&= -14(-1) \\
&= 14
\end{align*}
\]

b. \(\sqrt{-10} \cdot \sqrt{-15} \)
\[
\begin{align*}
\sqrt{-10} \cdot \sqrt{-15} &= i\sqrt{10} \cdot i\sqrt{15} \\
&= i^2\sqrt{150} \\
&= -1 \cdot \sqrt{25} \cdot \sqrt{6} \\
&= -5\sqrt{6}
\end{align*}
\]

c. \(i^{45} \)
\[
\begin{align*}
i^{45} &= i \cdot i^{44} \\
&= i \cdot (i^2)^{22} \\
&= i \cdot (-1)^{22} \\
&= i \cdot 1 \text{ or } i \\
&= i \\
&= i \cdot (-1)^{22} \\
&= -1
\end{align*}
\]

3A. \(3i \cdot 4i \)
\[
3i \cdot 4i = 12i^2 = 12(-1) = -12
\]

3B. \(\sqrt{-20} \cdot \sqrt{-12} \)
\[
\begin{align*}
\sqrt{-20} \cdot \sqrt{-12} &= \sqrt{-1 \cdot 20} \cdot \sqrt{-1 \cdot 12} \\
&= i\sqrt{20} \cdot i\sqrt{12} \\
&= i^2\sqrt{240} \\
&= -1 \cdot \sqrt{4^2} \cdot \sqrt{3} \\
&= -4\sqrt{3}
\end{align*}
\]

3C. \(i^{31} \)
\[
\begin{align*}
i^{31} &= i \cdot i^{30} \\
&= i \cdot (i^2)^{15} \\
&= i \cdot (-1)^{15} \\
&= i \cdot -1 \\
&= -i
\end{align*}
\]

Reading Math

Plus or Minus \( \pm \sqrt{n} \) is read plus or minus the square root of \( n \).

KEY CONCEPT

Square Root Property

For any real number \( n \), if \( x^2 = n \), then \( x = \pm \sqrt{n} \).

EXAMPLE

Equation with Pure Imaginary Solutions

Solve \( 3x^2 + 48 = 0 \).
\[
\begin{align*}
3x^2 + 48 &= 0 \quad \text{Original equation} \\
3x^2 &= -48 \quad \text{Subtract 48 from each side.} \\
x^2 &= -16 \quad \text{Divide each side by 3.} \\
x &= \pm \sqrt{-16} \quad \text{Square Root Property} \\
x &= \pm 4i
\end{align*}
\]

You can solve some quadratic equations by using the Square Root Property.
Operations with Complex Numbers Consider $5 + 2i$. Since 5 is a real number and $2i$ is a pure imaginary number, the terms are not like terms and cannot be combined. This type of expression is called a complex number.

The Venn diagram shows the complex numbers:
- If $b = 0$, the complex number is a real number.
- If $b \neq 0$, the complex number is imaginary.
- If $a = 0$, the complex number is a pure imaginary number.

Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal. That is, $a + bi = c + di$ if and only if $a = c$ and $b = d$.

EXAMPLE Equate Complex Numbers

Find the values of $x$ and $y$ that make the equation $2x - 3 + (y - 4)i = 3 + 2i$ true.

Set the real parts equal to each other and the imaginary parts equal to each other.

$2x - 3 = 3$ Real parts \[ y - 4 = 2 \] Imaginary parts

$2x = 6$ Add 3 to each side. \[ y = 6 \] Add 4 to each side.

$x = 3$ Divide each side by 2.

CHECK Your Progress 5. Find the values of $x$ and $y$ that make the equation $5x + 1 + (3 + 2y)i = 2x - 2 + (y - 6)i$ true.

To add or subtract complex numbers, combine like terms. That is, combine the real parts and combine the imaginary parts.
EXAMPLE Add and Subtract Complex Numbers

Simplify.

a. \((6 - 4i) + (1 + 3i)\)
   \[(6 - 4i) + (1 + 3i) = (6 + 1) + (-4 + 3)i\] Commutative and Associative Properties
   \[= 7 - i\] Simplify.
   
   b. \((3 - 2i) - (5 - 4i)\)
   \[(3 - 2i) - (5 - 4i) = (3 - 5) + [-2 - (-4)]i\] Commutative and Associative Properties
   \[= -2 + 2i\] Simplify.

6A. \((-2 + 5i) + (1 - 7i)\)  
6B. \((4 + 6i) - (-1 + 2i)\)

Complex Numbers
While all real numbers are also complex, the term Complex Numbers usually refers to a number that is not real.

ALGEBRA LAB

Adding Complex Numbers Graphically

Use a complex plane to find \((4 + 2i) + (-2 + 3i)\).
- Graph \(4 + 2i\) by drawing a segment from the origin to \((4, 2)\) on the complex plane.
- Graph \(-2 + 3i\) by drawing a segment from the origin to \((-2, 3)\) on the complex plane.
- Given three vertices of a parallelogram, complete the parallelogram.
- The fourth vertex at \((2, 5)\) represents the complex number \(2 + 5i\).

So, \((4 + 2i) + (-2 + 3i) = 2 + 5i\).

MODEL AND ANALYZE

1. Model \((-3 + 2i) + (4 - i)\) on a complex plane.
2. Describe how you could model the difference \((-3 + 2i) - (4 - i)\) on a complex plane.

Complex numbers are used with electricity. In a circuit with alternating current, the voltage, current, and impedance, or hindrance to current, can be represented by complex numbers. To multiply these numbers, use the FOIL method.
**Lesson 5-4 Complex Numbers**

**ELECTRICITY** In an AC circuit, the voltage $E$, current $I$, and impedance $Z$ are related by the formula $E = I \cdot Z$. Find the voltage in a circuit with current $1 + 3j$ amps and impedance $7 - 5j$ ohms.

$E = I \cdot Z$

Electricity formula

$E = (1 + 3j) \cdot (7 - 5j)$

$= (1)(7) + (1)(-5j) + (3j)(7) + (3j)(-5j)$

$= 7 - 5j + 21j - 15j^2$

$= 7 + 16j - 15(-1)$

$= 22 + 16j$

The voltage is $22 + 16j$ volts.

**CHECK Your Progress**

7. Find the voltage in a circuit with current $2 - 4j$ amps and impedance $3 - 2j$ ohms.

Two complex numbers of the form $a + bi$ and $a - bi$ are called **complex conjugates**. The product of complex conjugates is always a real number. You can use this fact to simplify the quotient of two complex numbers.

**EXAMPLE**

**Divide Complex Numbers**

**a.** Simplify.

$$\frac{3i}{2 + 4i}$$

$$= \frac{3i}{2 + 4i} \cdot \frac{2 - 4i}{2 - 4i}$$

$2 + 4i$ and $2 + 4i$ are conjugates.

$$= \frac{6i - 12i^2}{4 - 16i^2}$$

Multiply.

$$= \frac{6i + 12}{20}$$

$i^2 = -1$

$$= \frac{3}{5} + \frac{3}{10}i$$

Standard form

**b.** Simplify.

$$\frac{5 + i}{2i}$$

$$= \frac{5 + i}{2i} \cdot \frac{i}{i}$$

Why multiply by $\frac{i}{i}$ instead of $\frac{-2i}{-2i}$?

$$= \frac{5i + i^2}{2i^2}$$

Multiply.

$$= \frac{5i - 1}{-2}$$

$i^2 = -1$

$$= \frac{1}{2} - \frac{5}{2}i$$

Standard form

**8A.** Simplify.

$$\frac{-2i}{3 + 5i}$$

**8B.** Simplify.

$$\frac{2 + i}{1 - i}$$
Exercises

Examples 1–3 (pp. 259–260)

1. \(\sqrt{56}\)
2. \(\sqrt{80}\)
3. \(\sqrt{48}\)
4. \(\sqrt{120}\)
5. \(\sqrt{-36}\)
6. \(\sqrt{-50x^2y^2}\)
7. \((6i)(-2i)\)
8. \(5\sqrt{-24} \cdot 3\sqrt{-18}\)
9. \(i^{29}\)

Example 4 (p. 260)

Solve each equation.

11. \(2x^2 + 18 = 0\)
12. \(-5x^2 - 25 = 0\)

Example 5 (p. 261)

Find the values of \(m\) and \(n\) that make each equation true.

13. \(2m + (3n + 1)i = 6 - 8i\)
14. \((2n - 5) + (-m - 2)i = 3 - 7i\)

Example 6 (p. 262)

15. **ELECTRICITY** The current in one part of a series circuit is \(4 - j\) amps. The current in another part of the circuit is \(6 + 4j\) amps. Add these complex numbers to find the total current in the circuit.

Examples 7, 8 (p. 263)

Simplify.

16. \((-2 + 7i) + (-4 - 5i)\)
17. \((8 + 6i) - (2 + 3i)\)
18. \((3 - 5i)(4 + 6i)\)
19. \((1 + 2i)(-1 + 4i)\)
20. \(\frac{2 - i}{5 + 2i}\)
21. \(\frac{3 + i}{1 + 4i}\)

**Exercises**

**HOMEWORK**

<table>
<thead>
<tr>
<th>For Exercises</th>
<th>See Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>22–25</td>
<td>1</td>
</tr>
<tr>
<td>26–29</td>
<td>2</td>
</tr>
<tr>
<td>30–33</td>
<td>3</td>
</tr>
<tr>
<td>34–37</td>
<td>6</td>
</tr>
<tr>
<td>38, 39, 50</td>
<td>7</td>
</tr>
<tr>
<td>40, 41, 51</td>
<td>8</td>
</tr>
<tr>
<td>42–45</td>
<td>4</td>
</tr>
<tr>
<td>46–49</td>
<td>5</td>
</tr>
</tbody>
</table>

Simplify.

22. \(\sqrt{125}\) 
23. \(\sqrt{147}\) 
24. \(\sqrt{\frac{192}{121}}\) 
25. \(\sqrt{\frac{350}{81}}\) 
26. \(\sqrt{-144}\) 
27. \(\sqrt{-81}\) 
28. \(\sqrt{-64x^4}\) 
29. \(\sqrt{-100a^4b^2}\) 
30. \((-2i)(-6i)(4i)\) 
31. \(3i(-5i)^2\) 
32. \(i^{13}\) 
33. \(i^{24}\) 
34. \((5 - 2i) + (4 + 4i)\) 
35. \((-2 + i) + (-1 - i)\) 
36. \((15 + 3i) - (9 - 3i)\) 
37. \((3 - 4i) - (1 - 4i)\) 
38. \((3 + 4i)(3 - 4i)\) 
39. \((1 - 4i)(2 + i)\) 
40. \(\frac{4i}{3 + i}\) 
41. \(\frac{4}{5 + 3i}\)

Solve each equation.

42. \(5x^2 + 5 = 0\)
43. \(4x^2 + 64 = 0\)
44. \(2x^2 + 12 = 0\)
45. \(6x^2 + 72 = 0\)

Find the values of \(m\) and \(n\) that make each equation true.

46. \(8 + 15i = 2m + 3ni\)
47. \((m + 1) + 3ni = 5 - 9i\)
48. \((2m + 5) + (1 - n)i = -2 + 4i\)
49. \((4 + n) + (3m - 7)i = 8 - 2i\)

**ELECTRICITY** For Exercises 50 and 51, use the formula \(E = I \cdot Z\).

50. The current in a circuit is \(2 + 5j\) amps, and the impedance is \(4 - j\) ohms. What is the voltage?
51. The voltage in a circuit is $14 - 8j$ volts, and the impedance is $2 - 3j$ ohms. What is the current?

52. Find the sum of $ix^2 - (2 + 3i)x + 2$ and $4x^2 + (5 + 2i)x - 4i$.

53. Simplify $[(3 + i)x^2 - ix + 4 + i] - [(-2 + 3i)x^2 + (1 - 2i)x - 3]$.

54. Simplify

\[ \sqrt{-13} \cdot \sqrt{-26} \]

55. Simplify

\[ (4i) \left( \frac{1}{2}i \right)^2 (-2i)^2 \]

56. Simplify

\[ i^{38} \]

57. Simplify

\[ (3 - 5i) + (3 + 5i) \]

58. Simplify

\[ (7 - 4i) - (3 + i) \]

59. Simplify

\[ (-3 - i)(2 - 2i) \]

60. Simplify

\[ \frac{(10 + i)^2}{4 - i} \]

61. Simplify

\[ \frac{2 - i}{3 - 4i} \]

62. Simplify

\[ (-5 + 2i)(6 - i)(4 + 3i) \]

63. Simplify

\[ (2 + i)(1 + 2i)(3 - 4i) \]

64. Simplify

\[ \frac{5 - i\sqrt{3}}{5 + i\sqrt{3}} \]

65. Simplify

\[ \frac{1 - i\sqrt{2}}{1 + i\sqrt{2}} \]

Solve each equation, and locate the complex solutions in the complex plane.

66. Simplify

\[ -3x^2 - 9 = 0 \]

67. Simplify

\[ -2x^2 - 80 = 0 \]

68. Simplify

\[ \frac{2}{3}x^2 + 30 = 0 \]

69. Simplify

\[ \frac{4}{5}x^2 + 1 = 0 \]

Find the values of $m$ and $n$ that make each equation true.

70. Simplify

\[ (m + 2n) + (2m - n)i = 5 + 5i \]

71. Simplify

\[ (2m - 3n)i + (m + 4n) = 13 + 7i \]

72. **ELECTRICITY** The impedance in one part of a series circuit is $3 + 4j$ ohms, and the impedance in another part of the circuit is $2 - 6j$. Add these complex numbers to find the total impedance in the circuit.

73. **OPEN ENDED** Write two complex numbers with a product of 10.

74. **CHALLENGE** Copy and complete the table. Explain how to use the exponent to determine the simplified form of any power of $i$.

<table>
<thead>
<tr>
<th>Power of $i$</th>
<th>Simplified Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i^0$</td>
<td>?</td>
</tr>
<tr>
<td>$i^1$</td>
<td>?</td>
</tr>
<tr>
<td>$i^2$</td>
<td>?</td>
</tr>
<tr>
<td>$i^3$</td>
<td>?</td>
</tr>
<tr>
<td>$i^4$</td>
<td>?</td>
</tr>
<tr>
<td>$i^5$</td>
<td>?</td>
</tr>
<tr>
<td>$i^6$</td>
<td>?</td>
</tr>
<tr>
<td>$i^7$</td>
<td>?</td>
</tr>
<tr>
<td>$i^8$</td>
<td>?</td>
</tr>
<tr>
<td>$i^9$</td>
<td>?</td>
</tr>
<tr>
<td>$i^{10}$</td>
<td>?</td>
</tr>
<tr>
<td>$i^{11}$</td>
<td>?</td>
</tr>
<tr>
<td>$i^{12}$</td>
<td>?</td>
</tr>
<tr>
<td>$i^{13}$</td>
<td>?</td>
</tr>
</tbody>
</table>

75. **Which One Doesn’t Belong?** Identify the expression that does not belong with the other three. Explain your reasoning.

\[ (3i)^2 \quad (2i)(3i)(4i) \quad (6 + 2i) - (4 + 2i) \quad (2i)^4 \]

76. **REASONING** Determine if each statement is true or false. If false, find a counterexample.

a. Every real number is a complex number.

b. Every imaginary number is a complex number.
77. Writing in Math  Use the information on page 261 to explain how complex numbers are related to quadratic equations. Explain how the \(a\) and \(c\) must be related if the equation \(ax^2 + c = 0\) has complex solutions and give the solutions of the equation \(2x^2 + 2 = 0\).

78. ACT/SAT  The area of the square is 16 square units. What is the area of the circle?
A 2\(\pi\) units\(^2\)
B 12 units\(^2\)
C 4\(\pi\) units\(^2\)
D 16\(\pi\) units\(^2\)

79. If \(i^2 = -1\), then what is the value of \(i^{71}\)?
F 1
G 0
H \(-i\)
J \(i\)

Write a quadratic equation with the given root(s). Write the equation in the form \(ax^2 + bx + c = 0\), where \(a\), \(b\), and \(c\) are integers. (Lesson 5-3)

80. \(-3, 9\)
81. \(-\frac{1}{3}, -\frac{3}{4}\)

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. (Lesson 5-2)

82. \(3x^2 = 4 - 8x\)
83. \(2x^2 + 11x = -12\)

Triangle \(ABC\) is reflected over the \(x\)-axis. (Lesson 4-4)

84. Write a vertex matrix for the triangle.
85. Write the reflection matrix.
86. Write the vertex matrix for \(\triangle A'B'C'\).
87. Graph \(\triangle A'B'C'\).

88. FURNITURE  A new sofa, love seat, and coffee table cost $2050. The sofa costs twice as much as the love seat. The sofa and the coffee table together cost $1450. How much does each piece of furniture cost? (Lesson 3-5)

89. DECORATION  Samantha is going to use more than 75 but less than 100 bricks to make a patio off her back porch. If each brick costs $2.75, write and solve a compound inequality to determine the amount she will spend on bricks. (Lesson 1-6)

Determine whether each polynomial is a perfect square trinomial. (Lesson 5-3)

90. \(x^2 - 10x + 16\)
91. \(x^2 + 18x + 81\)
92. \(x^2 - 9\)
93. \(x^2 - 12x - 36\)
94. \(x^2 - x + \frac{1}{4}\)
95. \(2x^2 - 15x + 25\)
1. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex for \( f(x) = 3x^2 - 12x + 4 \). Then graph the function by making a table of values. (Lesson 5-1)

2. **MULTIPLE CHOICE** For which function is the x-coordinate of the vertex at 4? (Lesson 5-1)
   
   \[ a. f(x) = x^2 - 8x + 15 \]
   \[ b. f(x) = -x^2 - 4x + 12 \]
   \[ c. f(x) = x^2 + 6x + 8 \]
   \[ d. f(x) = -x^2 - 2x + 2 \]

3. Determine whether \( f(x) = 3 - x^2 + 5x \) has a maximum or minimum value. Then find this maximum or minimum value and state the domain and range of the function. (Lesson 5-1)

4. **BASEBALL** From 2 feet above home plate, Grady hits a baseball upward with a velocity of 36 feet per second. The height \( h(t) \) of the baseball \( t \) seconds after Grady hits it is given by \( h(t) = -16t^2 + 36t + 2 \). Find the maximum height reached by the baseball and the time that this height is reached. (Lesson 5-1)

5. Solve \( 2x^2 - 11x + 12 = 0 \) by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. (Lesson 5-2)

**NUMBER THEORY** Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist. (Lesson 5-2)

6. Their sum is 12, and their product is 20.

7. Their sum is 5 and their product is 9.

8. **MULTIPLE CHOICE** For what value of \( x \) does \( f(x) = x^2 + 5x + 6 \) reach its minimum value? (Lesson 5-2)
   
   \[ a. -5 \]
   \[ b. -\frac{5}{2} \]
   \[ c. -3 \]
   \[ d. -2 \]

9. **FOOTBALL** A place kicker kicks a ball upward with a velocity of 32 feet per second. Ignoring the height of the kicking tee, how long after the football is kicked does it hit the ground? Use the formula \( h(t) = v_0t - 16t^2 \) where \( h(t) \) is the height of an object in feet, \( v_0 \) is the object’s initial velocity in feet per second, and \( t \) is the time in seconds. (Lesson 5-2)

Solve each equation by factoring. (Lesson 5-3)

\[ a. 2x^2 - 5x - 3 = 0 \]
\[ b. 6x^2 + 4x - 2 = 0 \]
\[ c. 3x^2 - 6x - 24 = 0 \]
\[ d. x^2 + 12x + 20 = 0 \]

**REMODELING** For Exercises 14 and 15, use the following information. (Lesson 5-3)

Sandy’s closet was supposed to be 10 feet by 12 feet. The architect decided that this would not work and reduced the dimensions by the same amount \( x \) on each side. The area of the new closet is 63 square feet.

14. Write a quadratic equation that represents the area of Sandy’s closet now.

15. Find the new dimensions of her closet.

16. Write a quadratic equation in standard form with roots \(-4\) and \(\frac{1}{3}\). (Lesson 5-3)

Simplify. (Lesson 5-4)

\[ a. \sqrt{-49} \]
\[ b. \sqrt{-36a^3b^4} \]
\[ c. (28 - 4i) - (10 - 30i) \]
\[ d. i^{89} \]
\[ e. 2 - 4i \]
\[ f. \frac{1 + 3i}{1 + 3i} \]

23. **ELECTRICITY** The impedance in one part of a series circuit is \( 2 + 5j \) ohms and the impedance in another part of the circuit is \( 7 - 3j \) ohms. Add these complex numbers to find the total impedance in the circuit. (Lesson 5-4)

**Series Circuit**

<table>
<thead>
<tr>
<th>Number of Bulbs</th>
<th>Current</th>
<th>Brightness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.67</td>
<td>brightest</td>
</tr>
<tr>
<td>2</td>
<td>1.84</td>
<td>bright</td>
</tr>
<tr>
<td>3</td>
<td>1.22</td>
<td>dim</td>
</tr>
</tbody>
</table>
Under a yellow caution flag, race car drivers slow to a speed of 60 miles per hour. When the green flag is waved, the drivers can increase their speed.

Suppose the driver of one car is 500 feet from the finish line. If the driver accelerates at a constant rate of 8 feet per second squared, the equation \( t^2 + 22t + 121 = 246 \) represents the time \( t \) it takes the driver to reach this line. To solve this equation, you can use the Square Root Property.

**Square Root Property** You have solved equations like \( x^2 - 25 = 0 \) by factoring. You can also use the Square Root Property to solve such an equation. This method is useful with equations like the one above that describes the race car’s speed. In this case, the quadratic equation contains a perfect square trinomial set equal to a constant.

**EXAMPLE 1**

**Equation with Rational Roots**

Solve \( x^2 + 10x + 25 = 49 \) by using the Square Root Property.

\[
\begin{align*}
x^2 + 10x + 25 &= 49 & \text{Original equation} \\
(x + 5)^2 &= 49 & \text{Factor the perfect square trinomial.} \\
x + 5 &= \pm\sqrt{49} & \text{Square Root Property} \\
x + 5 &= \pm7 & \sqrt{49} = 7 \\
x &= -5 \pm 7 & \text{Add } -5 \text{ to each side.} \\
x &= -5 + 7 \quad \text{or} \quad x = -5 - 7 & \text{Write as two equations.} \\
x &= 2 & \text{Solve each equation.} \\
x &= -12 & \\
\end{align*}
\]

The solution set is \( \{2, -12\} \). You can check this result by using factoring to solve the original equation.

**CHECK Your Progress**

Solve each equation by using the Square Root Property.

1A. \( x^2 - 12x + 36 = 25 \) \hspace{1cm} 1B. \( x^2 - 16x + 64 = 49 \)

Roots that are irrational numbers may be written as exact answers in radical form or as approximate answers in decimal form when a calculator is used.
EXAMPLE  Equation with Irrational Roots

Solve \( x^2 - 6x + 9 = 32 \) by using the Square Root Property.

\[
\begin{align*}
  x^2 - 6x + 9 &= 32 & \text{Original equation} \\
  (x - 3)^2 &= 32 & \text{Factor the perfect square trinomial.} \\
  x - 3 &= \pm \sqrt{32} & \text{Square Root Property} \\
  x &= 3 \pm 4\sqrt{2} & \text{Add 3 to each side; } -\sqrt{32} = 4\sqrt{2} \\
  x &\approx 8.7 \quad \text{or} \quad x \approx -2.7 & \text{Write as two equations.}
\end{align*}
\]

The exact solutions of this equation are \( 3 - 4\sqrt{2} \) and \( 3 + 4\sqrt{2} \). The approximate solutions are \(-2.7\) and \(8.7\). Check these results by finding and graphing the related quadratic function.

CHECK Use the ZERO function of a graphing calculator. The approximate zeros of the related function are \(-2.7\) and \(8.7\).

Solve each equation by using the Square Root Property.

2A. \( x^2 + 8x + 16 = 20 \) \hspace{1cm} 2B. \( x^2 - 6x + 9 = 32 \)

Complete the Square  The Square Root Property can only be used to solve quadratic equations when the quadratic expression is a perfect square. However, few quadratic expressions are perfect squares. To make a quadratic expression a perfect square, a method called completing the square may be used.

In a perfect square trinomial, there is a relationship between the coefficient of the linear term and the constant term. Consider the following pattern.

\[
(x + 7)^2 = x^2 + 2(7)x + 7^2 \quad \text{Square of a sum pattern} \\
= x^2 + 14x + 49 \quad \text{Simplify.} \\
\downarrow \\
\left(\frac{14}{2}\right)^2 \rightarrow 7^2 \quad \text{Notice that 49 is } 7^2 \text{ and 7 is one half of 14.}
\]

Use this pattern of coefficients to complete the square of a quadratic expression.

KEY CONCEPT  Completing the Square

<table>
<thead>
<tr>
<th>Words</th>
<th>To complete the square for any quadratic expression of the form ( x^2 + bx ), follow the steps below.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>Find one half of ( b ), the coefficient of ( x ).</td>
</tr>
<tr>
<td>Step 2</td>
<td>Square the result in Step 1.</td>
</tr>
<tr>
<td>Step 3</td>
<td>Add the result of Step 2 to ( x^2 + bx ).</td>
</tr>
</tbody>
</table>

Symbols \( x^2 + bx + \left(\frac{b}{2}\right)^2 = x + \left(\frac{b}{2}\right)^2 \)
### EXAMPLE

**Complete the Square**

Find the value of $c$ that makes $x^2 + 12x + c$ a perfect square. Then write the trinomial as a perfect square.

1. **Step 1** Find one half of 12. \( \frac{12}{2} = 6 \)
2. **Step 2** Square the result of Step 1. \( 6^2 = 36 \)
3. **Step 3** Add the result of Step 2 to $x^2 + 12x$. \( x^2 + 12x + 36 \)

The trinomial $x^2 + 12x + 36$ can be written as $(x + 6)^2$.

### Check Your Progress

3. Find the value of $c$ that makes $x^2 - 14x + c$ a perfect square. Then write the trinomial as a perfect square.

You can solve any quadratic equation by completing the square. Because you are solving an equation, add the value you use to complete the square to each side.

### ALGEBRA LAB

**Completing the Square**

Use algebra tiles to complete the square for the equation $x^2 + 2x - 3 = 0$.

1. **Step 1** Represent $x^2 + 2x - 3 = 0$ on an equation mat.
   
   ![Equation Mat](image)

   \[ x^2 + 2x - 3 = 0 \]

2. **Step 2** Add 3 to each side of the mat. Remove the zero pairs.
   
   ![Zero Pairs Removed](image)

   \[ x^2 + 2x + 3 = 0 + 3 \]

3. **Step 3** Begin to arrange the $x^2$- and $x$-tiles into a square.
   
   ![Arranging Tiles](image)

   \[ x^2 + 2x = 3 \]

4. **Step 4** To complete the square, add 1 yellow 1-tile to each side. The completed equation is $x^2 + 2x + 1 = 4$ or $(x + 1)^2 = 4$.
   
   ![Completed Square](image)

   \[ x^2 + 2x + 1 = 3 + 1 \]

**MODEL**

Use algebra tiles to complete the square for each equation.

1. $x^2 + 2x - 4 = 0$
2. $x^2 + 4x + 1 = 0$
3. $x^2 - 6x = -5$
4. $x^2 - 2x = -1$
Lesson 5-5 Completing the Square

**Common Misconception**
When solving equations by completing the square, don’t forget to add \((b/2)^2\) to each side of the equation.

**EXAMPLE**

**Solve an Equation by Completing the Square**

4. Solve \(x^2 + 8x - 20 = 0\) by completing the square.

\[
\begin{align*}
x^2 + 8x - 20 &= 0 & \text{Notice that } x^2 + 8x - 20 \text{ is not a perfect square.} \\
x^2 + 8x &= 20 & \text{Rewrite so the left side is of the form } x^2 + bx. \\
x^2 + 8x + 16 &= 20 + 16 & \text{Since } \left(\frac{8}{2}\right)^2 = 16, \text{ add 16 to each side.} \\
(x + 4)^2 &= 36 & \text{Write the left side as a perfect square by factoring.} \\
x + 4 &= \pm 6 & \text{Add \(-4\) to each side.} \\
x &= -4 \pm 6 & \text{Write as two equations.} \\
x &= 2 \quad \text{or} \quad x &= -4 - 6 \\
x &= -10 & \text{The solution set is } \{-10, 2\}. 
\end{align*}
\]

You can check this result by using factoring to solve the original equation.

**CHECK Your Progress**

Solve each equation by completing the square.

4A. \(x^2 - 10x + 24 = 0\) \quad 4B. \(x^2 + 10x + 9 = 0\)

When the coefficient of the quadratic term is not 1, you must divide the equation by that coefficient before completing the square.

**EXAMPLE**

**Equation with \(a \neq 1\)**

5. Solve \(2x^2 - 5x + 3 = 0\) by completing the square.

\[
\begin{align*}
2x^2 - 5x + 3 &= 0 & \text{Notice that } 2x^2 - 5x + 3 \text{ is not a perfect square.} \\
x^2 - \frac{5}{2}x + \frac{3}{2} &= 0 & \text{Divide by the coefficient of the quadratic term, } 2. \\
x^2 - \frac{5}{2}x &= -\frac{3}{2} & \text{Subtract } \frac{3}{2} \text{ from each side.} \\
x^2 - \frac{5}{2}x + \frac{25}{16} &= -\frac{3}{2} + \frac{25}{16} & \text{Since } \left(\frac{-\frac{5}{2} + \frac{1}{2}}{2}\right)^2 = \frac{25}{16}, \text{ add } \frac{25}{16} \text{ to each side.} \\
\left(x - \frac{5}{4}\right)^2 &= \frac{1}{16} & \text{Write the left side as a perfect square by factoring.} \\
x - \frac{5}{4} &= \pm \frac{1}{4} & \text{Simplify the right side.} \\
x &= \frac{5}{4} \pm \frac{1}{4} & \text{Add } \frac{5}{4} \text{ to each side.} \\
x &= \frac{3}{2} \quad \text{or} \quad x = \frac{5}{4} - \frac{1}{4} & \text{Write as two equations.} \\
x &= \frac{3}{2} & \text{The solution set is } \left\{1, \frac{3}{2}\right\}. 
\end{align*}
\]

**CHECK Your Progress**

Solve each equation by completing the square.

5A. \(3x^2 + 10x - 8 = 0\) \quad 5B. \(3x^2 - 14x + 16 = 0\)
Not all solutions of quadratic equations are real numbers. In some cases, the solutions are complex numbers of the form \( a + bi \), where \( b \neq 0 \).

**EXAMPLE**  
**Equation with Complex Solutions**

Solve \( x^2 + 4x + 11 = 0 \) by completing the square.

\[
x^2 + 4x + 11 = 0 \quad \text{Notice that } x^2 + 4x + 11 \text{ is not a perfect square.}
\]

\[
x^2 + 4x = -11 \quad \text{Rewrite so the left side is of the form } x^2 + bx.
\]

\[
x^2 + 4x + 4 = -11 + 4 \quad \text{Since } (\frac{4}{2})^2 = 4, \text{ add } 4 \text{ to each side.}
\]

\[
(x + 2)^2 = -7 \quad \text{Write the left side as a perfect square by factoring.}
\]

\[
x + 2 = \pm \sqrt{-7} \quad \text{Square Root Property}
\]

\[
x + 2 = \pm i\sqrt{7} \quad \sqrt{-1} = i
\]

\[
x = -2 \pm i\sqrt{7} \quad \text{Subtract } 2 \text{ from each side.}
\]

The solution set is \( \{-2 + i\sqrt{7}, -2 - i\sqrt{7}\} \). Notice that these are imaginary solutions.

**CHECK**  
A graph of the related function shows that the equation has no real solutions since the graph has no \( x \)-intercepts. Imaginary solutions must be checked algebraically by substituting them in the original equation.

**Solve each equation by completing the square.**

6A. \( x^2 + 2x + 2 = 0 \)

6B. \( x^2 - 6x + 25 = 0 \)

**Examples 1 and 2**

Solve each equation by using the Square Root Property.

1. \( x^2 + 14x + 49 = 9 \)
2. \( x^2 - 12x + 36 = 25 \)
3. \( x^2 + 16x + 64 = 7 \)
4. \( 9x^2 - 24x + 16 = 2 \)

**ASTRONOMY**  
For Exercises 5–7, use the following information.

The height \( h \) of an object \( t \) seconds after it is dropped is given by \( h = -\frac{1}{2}gt^2 + h_0 \), where \( h_0 \) is the initial height and \( g \) is the acceleration due to gravity. The acceleration due to gravity near Earth’s surface is 9.8 m/s\(^2\), while on Jupiter it is 23.1 m/s\(^2\). Suppose an object is dropped from an initial height of 100 meters from the surface of each planet.

5. On which planet should the object reach the ground first?
6. Find the time it takes for the object to reach the ground on each planet to the nearest tenth of a second.
7. Do the times to reach the ground seem reasonable? Explain.
Find the value of $c$ that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

8. $x^2 - 12x + c$
9. $x^2 - 3x + c$

Solve each equation by completing the square.

10. $x^2 + 3x - 18 = 0$
11. $x^2 - 8x + 11 = 0$
12. $2x^2 - 3x - 3 = 0$
13. $3x^2 + 12x - 18 = 0$
14. $x^2 + 2x + 6 = 0$
15. $x^2 - 6x + 12 = 0$

Find the value of $c$ that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

16. $x^2 + 4x + 4 = 25$
17. $x^2 - 10x + 25 = 49$
18. $x^2 - 9x + \frac{81}{4} = \frac{1}{4}$
19. $x^2 + 7x + \frac{49}{4} = 4$
20. $x^2 + 8x + 16 = 7$
21. $x^2 - 6x + 9 = 8$
22. $x^2 + 12x + 36 = 5$
23. $x^2 - 3x + \frac{9}{4} = 6$

Solve each equation by using the Square Root Property.

24. $x^2 + 16x + c$
25. $x^2 - 18x + c$
26. $x^2 - 15x + c$
27. $x^2 + 7x + c$
28. $x^2 - 8x + 15 = 0$
29. $x^2 + 2x - 120 = 0$
30. $x^2 + 2x - 6 = 0$
31. $x^2 - 4x + 1 = 0$
32. $2x^2 + 3x - 5 = 0$
33. $2x^2 - 3x + 1 = 0$
34. $2x^2 + 7x + 6 = 0$
35. $9x^2 - 6x - 4 = 0$
36. $x^2 - 4x + 5 = 0$
37. $x^2 + 6x + 13 = 0$
38. $x^2 - 10x + 28 = 0$
39. $x^2 + 8x + 9 = -9$

40. **MOVIE SCREENS** The area $A$ in square feet of a projected picture on a movie screen is given by $A = 0.16d^2$, where $d$ is the distance from the projector to the screen in feet. At what distance will the projected picture have an area of 100 square feet?

41. **FRAMING** A picture has a square frame that is 2 inches wide. The area of the picture is one third of the total area of the picture and frame. What are the dimensions of the picture to the nearest quarter of an inch?

Find the value of $c$ that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

42. $x^2 + x + \frac{1}{4} = \frac{9}{16}$
43. $x^2 + 1.4x + 0.49 = 0.81$
44. $4x^2 - 28x + 49 = 5$
45. $9x^2 + 30x + 25 = 11$

Solve each equation by completing the square.

46. $x^2 + 0.6x + c$
47. $x^2 - 2.4x + c$
48. $x^2 - \frac{8}{3}x + c$
49. $x^2 + \frac{5}{2}x + c$

Find the value of $c$ that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

50. $x^2 + 1.4x = 1.2$
51. $x^2 - 4.7x = -2.8$
52. $x^2 - \frac{2}{3}x - \frac{26}{9} = 0$
53. $x^2 - \frac{3}{2}x - \frac{23}{16} = 0$
54. $3x^2 - 4x = 2$
55. $2x^2 - 7x = -12$
56. **ENGINEERING** In an engineering test, a rocket sled is propelled into a target. The sled’s distance \( d \) in meters from the target is given by the formula \( d = -1.5t^2 + 120 \), where \( t \) is the number of seconds after rocket ignition. How many seconds have passed since rocket ignition when the sled is 10 meters from the target?

57. **GOLDEN RECTANGLE** For Exercises 57–59, use the following information. A golden rectangle is one that can be divided into a square and a second rectangle that is geometrically similar to the original rectangle. The ratio of the length of the longer side to the shorter side of a golden rectangle is called the golden ratio.

58. Find the ratio of the length of the longer side to the length of the shorter side for rectangle \( AB \), and for rectangle \( EF \).

59. **RESEARCH** Use the Internet or other reference to find examples of the golden rectangle in architecture. What applications does the golden ratio have in music?

60. **KENNEL** A kennel owner has 164 feet of fencing with which to enclose a rectangular region. He wants to subdivide this region into three smaller rectangles of equal length, as shown. If the total area to be enclosed is 576 square feet, find the dimensions of the enclosed region. (Hint: Write an expression for \( l \) in terms of \( w \).)

61. **OPEN ENDED** Write a perfect square trinomial equation in which the linear coefficient is negative and the constant term is a fraction. Then solve the equation.

62. **FIND THE ERROR** Rashid and Tia are solving \( 2x^2 - 8x + 10 = 0 \) by completing the square. Who is correct? Explain your reasoning.

<table>
<thead>
<tr>
<th>Rashid</th>
<th>Tia</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2x^2 - 8x + 10 = 0 )</td>
<td>( 2x^2 - 8x + 10 = 0 )</td>
</tr>
<tr>
<td>( 2x^2 - 8x = -10 )</td>
<td>( x^2 - 4x = 0 - 5 )</td>
</tr>
<tr>
<td>( 2x^2 - 8x + 16 = -10 + 16 )</td>
<td>( x^2 - 4x + 4 = -5 + 4 )</td>
</tr>
<tr>
<td>( (x - 4)^2 = 6 )</td>
<td>( (x - 2)^2 = -1 )</td>
</tr>
<tr>
<td>( x - 4 = \pm \sqrt{6} )</td>
<td>( x = 2 \pm i )</td>
</tr>
</tbody>
</table>

63. **REASONING** Determine whether the value of \( c \) that makes \( ax^2 + bx + c \) a perfect square trinomial is sometimes, always, or never negative. Explain your reasoning.
64. **Challenge** Find all values of \( n \) such that \( x^2 + bx + \left(\frac{b}{2}\right)^2 = n \) has
   a. one real root.  
   b. two real roots.  
   c. two imaginary roots.

65. **Writing in Math** Use the information on page 268 to explain how you can find the time it takes an accelerating car to reach the finish line. Include an explanation of why \( t^2 + 22t + 121 = 246 \) cannot be solved by factoring and a description of the steps you would take to solve the equation.

### Standardized Test Practice

66. **ACT/SAT** The two zeros of a quadratic function are labeled \( x_1 \) and \( x_2 \) on the graph. Which expression has the greatest value?
   A. \( 2x_1 \)
   B. \( x_2 \)
   C. \( x_2 - x_1 \)
   D. \( x_2 + x_1 \)

67. **Review** If \( i = \sqrt{-1} \) which point shows the location of \( 2 - 4i \) on the plane?
   F. point A
   G. point B
   H. point C
   J. point D

### Spiral Review

Simplify. **(Lesson 5-4)**

68. \( i^{14} \)

69. \((4 - 3i) - (5 - 6i)\)

70. \((7 + 2i)(1 - i)\)

Solve each equation by factoring. **(Lesson 5-3)**

71. \(4x^2 + 8x = 0\)

72. \(x^2 - 5x = 14\)

73. \(3x^2 + 10 = 17x\)

Solve each system of equations by using inverse matrices. **(Lesson 4-8)**

74. \(5x + 3y = -5\)
   \(7x + 5y = -11\)

75. \(6x + 5y = 8\)
   \(3x - y = 7\)

### Chemistry

For Exercises 76 and 77, use the following information.
For hydrogen to be a liquid, its temperature must be within 2°C of \(-257°C\). **(Lesson 1-4)**

76. Write an equation to determine the least and greatest temperatures for this substance.

77. Solve the equation.

### Get Ready for the Next Lesson

**Prerequisite Skill** Evaluate \( b^2 - 4ac \) for the given values of \( a, b, \) and \( c \). **(Lesson 1-1)**

78. \(a = 1, b = 7, c = 3\)

79. \(a = 1, b = 2, c = 5\)

80. \(a = 2, b = -9, c = -5\)

81. \(a = 4, b = -12, c = 9\)